Multipole compensation of long-range beam-beam interactions with minimization of nonlinearities in Poincaré maps of a storage-ring collider

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In cases of multibunch operation in storage-ring colliders, serious long-range beam-beam effects could be due to many parasitic collisions that are localized inside interaction regions or/and distributed around the ring. To reduce the long-range beam-beam effects, the compensation of long-range beam-beam interaction with magnetic multipole correctors based on minimizations of nonlinearities in one-turn or sectional maps of a collider has been proposed. With Large Hadron Collider as a test model, the effectiveness of the multipole compensation of long-range beam-beam interactions was studied in terms of improvement of dynamic aperture and reduction of emittance growth. The emittance growth was studied with a self-consistent beam-beam simulation. The study showed that both the local and global compensation of long-range beam-beam interactions with magnetic multipole correctors are very effective in increasing the dynamic aperture and improving the linearity of the phase-space region relevant to the beams.

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I. INTRODUCTION

In large storage-ring colliders such as LHC (Large Hadron Collider being constructed in CERN, Switzerland) and Tevatron (Fermilab, Chicago), the long-range beam-beam interaction could be a major factor that reduces the beam lifetime and limits the luminosity. For LHC, a wire compensation scheme has been proposed to compensate the long-range beam-beam perturbations due to parasitic collisions inside interaction regions [1]. Simulation studies showed that the wire compensation is very effective to such localized long-range beam-beam perturbations [2,3]. In the case of multibunch operation in Tevatron RUN II, serious long-range beam-beam effects are due to many parasitic collisions that are distributed around the ring. For such nonlocalized long-range beam-beam perturbations, it is very difficult, if not impossible, to apply the wire compensation scheme. The electron-beam compensation of beam-beam tune spread, on the other hand, has been proposed for an elimination of bunch-to-bunch tune variation due to the parasitic collisions in Tevatron [4,5]. The nonlinear beam-beam perturbation from the parasitic collisions could still pose a serious problem that causes emittance growth and limits the luminosity even after the elimination of the tune variation. Note that the use of the electron-beam compensation to reduce the beam-beam tune spread within a bunch may not improve and could even damage the beam stability [6]. In order to control adverse effects of long-range beam-beam interactions in such cases involving a large number of nonlocalized parasitic collisions, we proposed a compensation scheme: the global or local compensation of long-range beam-beam interactions by using magnetic multipole correctors based on minimization of nonlinearities in one-turn or sectional maps of a storage-ring collider. This multipole compensation of long-range beam-beam interactions can be used either for a local compensation of localized parasitic collisions such as the case of LHC or for a global compensation of a large number of nonlocalized parasitic collisions such as the case of Tevatron.

The principle of the multipole compensation of long-range beam-beam interactions is based on the idea of global compensation of nonlinearity in a storage-ring collider by using maps [7,8]. The use of Poincaré (one-turn) maps to study nonlinear beam dynamics in storage rings has been developed in the last two decades by many researchers [9–11]. It is well understood that without beam-beam interactions, the nonlinear beam dynamics in a storage ring can be described by a one-turn map that contains all global information of nonlinearities in the system. By minimizing nonlinear terms of one-turn maps order by order with a few groups of multipole correctors, one can reduce the nonlinearity of the system globally. To include long-range beam-beam interactions into the map for the global compensation, one should recognize that a large beam separation is typical at parasitic collision points. In both LHC and Tevatron, for example, the beam separation is in a scale of $(6–14)\sigma$ [1,12], where σ is the nominal beam size. In the phase-space region relevant to the beam, the long-range beam-beam interactions can thus be expanded into a Taylor series around the beam separation and be included into the one-turn map for the global compensation of the nonlinearities of the system. In the case of localized parasitic collisions, one can use a group of local multipole correctors to minimize nonlinear terms of a local sectional map that contains the Taylor expansion of long-range beam-beam interactions [14]. With a few groups of multipoles correctors, therefore, nonlinear terms in one-turn maps and/or sectional maps including the long-range beam-beam interactions can be minimized order by order and, consequently, the nonlinearity of the system in the phase-space region of interest can be significantly reduced.

With LHC as a model, the effectiveness of the multipole compensation of long-range beam-beam interactions was evaluated in terms of the improvement of dynamic aperture and the reduction of emittance growth. Dynamic aperture of

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LHC with and without the multipole compensation was calculated with 10^5-turn tracking. Head-on and long-range beam-beam interactions at two high-luminosity interaction points (IPs) as well as multipole field errors in the lattice were included in the tracking. The emittance growth was examined with a self-consistent beam-beam simulation by using the particle-in-cell method with 5×10^5 particles in each beam. The study showed that the multipole compensation of long-range beam-beam interactions is very effective in increasing the dynamic aperture and improving the linearity of the phase-space region relevant to the beams. This paper is organized as follows. In Sec. II, the principle of the multipole compensation of long-range beam-beam interactions is discussed. The testing result of the multipole compensation in LHC is presented in Sec. III. Section V contains a summary.

II. MULTIPOLE COMPENSATION SCHEME

In high-energy storage-ring colliders, long-range beam-beam interactions at parasitic crossings can be approximated by momentum kicks in transverse phase space [13]. Consider the beam that exerts the beam-beam interaction is a round Gaussian beam at IPs. The momentum kick on a particle in the counterrotating beam can be calculated from [13]

$$\Delta\vec{p}=\frac{G_0}{2\pi\sigma_x\sigma_y}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\vec{r}+\vec{r}_0-\vec{r}_1}{|\vec{r}+\vec{r}_0-\vec{r}_1|^2}\times\exp\left(-\frac{x_1^2}{2\sigma_x^2}-\frac{y_1^2}{2\sigma_y^2}\right)dx_1dy_1, \qquad (1)$$

where $\vec{r}=(x,y)$ are transverse coordinates of the particle and $\vec{p}=(p_x,p_y)$ the conjugate momenta. $\vec{r}_0=(x_0,y_0)$ and (σ_x,σ_y) are the horizontal and vertical beam separation and rms size of the beam that exerts the kick at the parasitic crossing, respectively. The kick strength G_0 is related to the beam-beam parameter ξ by $G_0=8\pi\sigma^{*2}\xi/\beta^*$, where σ^* and β^* are the nominal beam size and beta function at IP. By using $1/a=\int_0^\infty e^{-at}dt$, the integrals in Eq. (1) can be changed into

$$\Delta p_x=\frac{G_0(x+x_0)}{2\sigma_x^2}\times\int_0^1 d\tau f_x\exp\left\{-\frac{\tau}{2\sigma_x^2}[(x+x_0)^2+f_x^2(y+y_0)^2]\right\}, \qquad (2)$$

$$\Delta p_y=\frac{G_0(y+y_0)}{2\sigma_y^2}\times\int_0^1 d\tau f_y\exp\left\{-\frac{\tau}{2\sigma_y^2}[(y+y_0)^2+f_y^2(x+x_0)^2]\right\}, \qquad (3)$$

where $f_x(\tau)=[1+(\sigma_y^2/\sigma_x^2-1)\tau]^{-1/2}$ and $f_y(\tau)=[1+(\sigma_x^2/\sigma_y^2-1)\tau]^{-1/2}$. Note that at parasitic crossings r_0 is much larger than the size of the beams and therefore $r_0\gg r$ in Eqs. (1)–(3). The beam separation could, however, be in any direction and it is not always true that $x_0\gg x$ and $y_0\gg y$. $\Delta\vec{p}$ in Eqs. (2) and (3) cannot be directly expanded into polynominals of x and y at x_0 and y_0. In order to guarantee the convergence of the expansion, $\Delta\vec{p}$ should be expanded in the direction of the beam separation at each parasitic crossing point. $\Delta\vec{p}$ in Eqs. (2) and (3) was therefore rewritten as

$$\Delta p_x=\frac{G_0(x+x_0)}{2\sigma_x^2}\int_0^1 d\tau f_x\exp\left[-\frac{\tau(1-f_x^2)}{2\sigma_x^2}(x+x_0)^2\right]\times\exp\left[-\frac{\tau f_x^2}{2\sigma_x^2}|\vec{r}+\vec{r}_0|^2\right] \qquad (4)$$

when $x_0\geqslant y_0$, and

$$\Delta p_x=\frac{G_0(x+x_0)}{2\sigma_x^2}\int_0^1 d\tau f_x\exp\left[-\frac{\tau(f_x^2-1)}{2\sigma_x^2}(y+y_0)^2\right]\times\exp\left[-\frac{\tau}{2\sigma_x^2}|\vec{r}+\vec{r}_0|^2\right] \qquad (5)$$

when $y_0>x_0$. Similar equations for Δp_y can be obtained by exchanging x and y and exchanging the conditions of $x_0>y_0$ and $y_0>x_0$ in Eqs. (4) and (5). When $x_0>y_0$, $x_0\gg x$ and, therefore, $(x+x_0)^2-x_0^2\ll1$ and $|\vec{r}+\vec{r}_0|^2-r_0^2\ll1$. When $y_0>x_0$, $y_0\gg y$ and, therefore, $(y+y_0)^2-y_0^2\ll1$ and $|\vec{r}+\vec{r}_0|^2-r_0^2\ll1$. Δp_x can then be expanded into a Taylor series of $(x+x_0)^2-x_0^2$ and $|\vec{r}+\vec{r}_0|^2-r_0^2$ when $x_0\geqslant y_0$ or a Taylor series of $(y+y_0)^2-y_0^2$ and $|\vec{r}+\vec{r}_0|^2-r_0^2$ when $y_0>x_0$:

$$\Delta p_x=\frac{G_0(x+x_0)}{2\sigma_x^2}\sum_{n,m=0}^{m+n=M}\frac{(-1)^{n+m}C_{mn}^{(1)}}{n!m!(2\sigma_x^2)^{n+m}}[|\vec{r}+\vec{r}_0|^2-r_0^2]^m\times[(x+x_0)^2-x_0^2]^n+O(M+1) \qquad (6)$$

for $x_0\geqslant y_0$ where

$$C_{mn}^{(1)}=\int_0^1\tau^{n+m}f_x^{2m+1}(1-f_x^2)^n\exp\left[-\frac{\tau(x_0^2+f_x^2y_0^2)}{2\sigma_x}\right]d\tau,$$

and

$$\Delta p_x=\frac{G_0(x+x_0)}{2\sigma_x^2}\sum_{n,m=0}^{m+n=M}\frac{(-1)^{n+m}C_{mn}^{(2)}}{n!m!(2\sigma_x^2)^{n+m}}\times[|\vec{r}+\vec{r}_0|^2-r_0^2]^m[(y+y_0)^2-y_0^2]^n+O(M+1) \qquad (7)$$

for $y_0>x_0$, where

$$C_{mn}^{(2)}=\int_0^1\tau^{n+m}f_x(f_x^2-1)^n\exp\left[-\frac{\tau(x_0^2+f_x^2y_0^2)}{2\sigma_x}\right]d\tau.$$

In Eqs. (6) and (7), M is the order of the truncation of the expansions in the calculation of one-turn or sectional maps. The neglected terms $O(M+1)$ are of the order of $[|\vec{r}+\vec{r}_0|^2-r_0^2]^m[(x+x_0)^2-x_0^2]^n$ in Eq. (6) or $[|\vec{r}+\vec{r}_0|^2-r_0^2]^m[(y+y_0)^2-y_0^2]^n$ in Eq. (7), with $m+n=M+1$. Similarly, Δp_y can be expanded as

$$\Delta p_y=\frac{G_0(y+y_0)}{2\sigma_y^2}\sum_{n,m=0}^{m+n=M}\frac{(-1)^{n+m}D_{mn}^{(1)}}{n!m!(2\sigma_y^2)^{n+m}}\times[|\vec{r}+\vec{r}_0|^2-r_0^2]^m[(y+y_0)^2-y_0^2]^n+O(M+1) \tag{8}$$

for $y_0\geqslant x_0$, where

$$D_{mn}^{(1)}=\int_0^1\tau^{n+m}f_y^{2m+1}(1-f_y^2)^n\exp\left[-\frac{\tau(y_0^2+f_y^2x_0^2)}{2\sigma_y}\right]d\tau,$$

and

$$\Delta p_y=\frac{G_0(y+y_0)}{2\sigma_y^2}\sum_{n,m=0}^{m+n=M}\frac{(-1)^{n+m}D_{mn}^{(2)}}{n!m!(2\sigma_y^2)^{n+m}}\times[|\vec{r}+\vec{r}_0|^2-r_0^2]^m[(x+x_0)^2-x_0^2]^n+O(M+1) \tag{9}$$

for $x_0>y_0$, where

$$D_{mn}^{(2)}=\int_0^1\tau^{n+m}f_y(f_y^2-1)^n\exp\left[-\frac{\tau(y_0^2+f_y^2x_0^2)}{2\sigma_y}\right]d\tau.$$

At any parasitic crossing, $C_{mn}^{(1)}$, $C_{mn}^{(2)}$, $D_{mn}^{(1)}$, and $D_{mn}^{(2)}$ in Eqs. (6)–(9) can be calculated by using numerical integration. With the method of differential algebra (automatic differentiation) [10] or Lie algebra [9], $\Delta\vec{p}$ as polynominals of x and y can be calculated by using Eqs. (6)–(9) to any desired order. Neglecting the transverse and longitudinal coupling, a four-dimensional one-turn or sectional map for the transverse motion including long-range beam-beam interactions can then be calculated in the form of Dragt-Finn Factorization [15]

$$M=R_0e^{:H_3:}e^{:H_4:}\cdots e^{:H_n:}\cdots, \tag{10}$$

where R_0 is a linear transformation that corresponds to the betatron oscillation. $:H_n:$ represents the Lie operator associated with the function $H_n(\vec{r},\vec{p})$, which is defined by the Poisson bracket operation $:H_n:\vec{r}=[H_n,\vec{r}]$. H_n is a homogeneous polynomial of degree n that is associated with the $(n-1)$th-order nonlinearity in the map,

$$H_n=\sum_{i+j+k+l=n}u_{ijkl}x^ip_x^jy^kp_y^l, \tag{11}$$

where u_{ijkl} are constant coefficients containing all global information of nonlinearities of the system, including the long-range beam-beam interactions and multipole field errors in the lattice. For an accelerator, since the phase-space region near the origin is of most interest, low-order nonlinearities of one-turn or sectional maps are usually more important than high-order terms of the map. The global or local compensation of the nonlinearities with the map is based on an assumption that with a few groups of multipole correctors, $\{H_n|n\geqslant3\}$, can be minimized order by order and, consequently, the dynamics of the system can be substantially improved. In order to minimize $\{H_n|n\geqslant3\}$ with a few parameters of the global correctors, we postulate that the nth-order undesirable nonlinearity in a one-turn or sectional map can be characterized by the magnitude of its nth-order undesirable coefficients which are defined by

$$\lambda_2=\left[\sum_{i+j+k+l=3}(u_{ijkl}-u_{ijkl}^0)^2\right]^{1/2} \tag{12}$$

and

$$\lambda_n=\left[\sum_{i+j+k+l=n+1}u_{ijkl}^2\right]^{1/2}\quad\text{for}\quad n>2, \tag{13}$$

where u_{ijkl}^0 of $i+j+k+l=3$ denote the sextupole components from the chromaticity correctors and should be excluded from the minimization of H_3. For convenience, we define the nth-order global/local compensation when all λ_i with $i=2,\ldots,n$ are minimized order by order using the multipole correctors up to the nth order. For example, the second-order compensation is to use sextupole correctors to minimize λ_2, the quadratic nonlinearity in the map.

To implement the global or local compensation of the nonlinearities with the one-turn or/and sectional maps, the maps can be obtained by using the method of differential algebra [10] or Lie algebra [9] based on the measured field errors of magnets and the long-range beam-beam interactions at designed parasitic beam crossings. During the operation, the beam-beam interactions and thus the maps change with the beam sizes. The strength of the multipole correctors for the compensation could be adjusted dynamically with the beam-size growth. If a one-turn map can be extracted with desired accuracy directly from beam-dynamics measurements [16,17], moreover, the global compensation of the nonlinearities could be further optimized during the commission of an accelerator [8]. Such a beam-based global compensation is especially important when there is a significant uncertainty in the linear lattice and/or in the field measurement of magnets.

III. TESTING RESULT OF MULTIPOLE COMPENSATION IN LHC

The test model used in this study is the LHC collision lattice [18]. The fractional parts of horizontal and vertical tunes are $(\nu_x,\nu_y)=(0.31,0.32)$. Head-on and long-range beam-beam interactions at two high-luminosity IPs as well as multipole field errors in the lattice were included in the study. For long-range beam-beam interactions, there are 15 parasitic crossings on the each side of an IP with vertical separation at one IP and horizontal separation at another. The separation of two beams at parasitic collisions ranges from

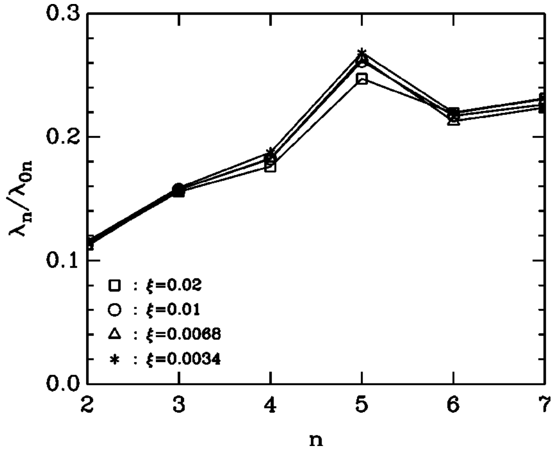

FIG. 1. The magnitude of one-turn map coefficients λ_n after the multipole compensation of long-range beam-beam interactions vs the order of the compensation. λ_{0n} is the magnitude of one-turn map coefficients without the compensation. The correctors for the multipole compensation are in the arcs. Four curves of ξ=0.0034, 0.0068, 0.01, and 0.02 almost overlap each other.

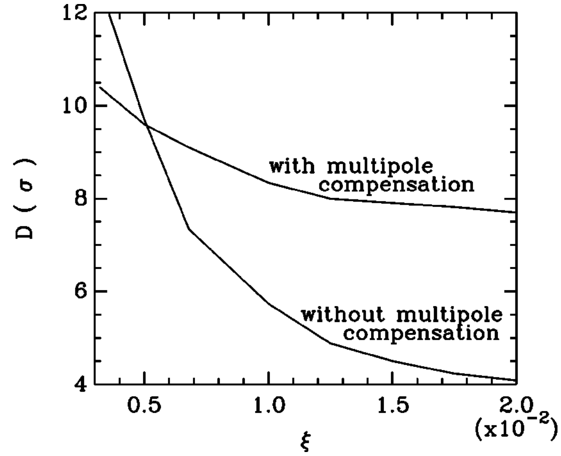

FIG. 2. Dynamic aperture (D) vs beam-beam parameter (ξ) without or with the multipole compensation of long-range beam-beam interactions in the linear lattice with head-on and long-range beam-beam interactions. σ is the nominal transverse beam size of LHC.

7σ to 13σ, where σ is the transverse beam size. In LHC, nonlinearities of the collision lattice are dominated by nonlinear magnetic field errors inside the inner triplets of interaction regions. Each triplet contains a group of multipole correctors for a correction of those nonlinear fields. To test the local compensation of both long-range beam-beam interactions and the nonlinear fields in the triplets, we used those correctors in the triplets to minimize λ_n. To test the global compensation of both long-range beam-beam interactions and nonlinear field errors in the lattice, we also included four corrector packages symmetrically located in arcs. Each package of correctors contains normal and skew components of desired multipole correctors. Before and after the compensation of long-range beam-beam interactions, the linear lattice in this study was always tuned to the design value.

In order to isolate the effect of the multipole compensation on long-range beam-beam interactions, we first consider the case that contains only head-on and long-range beam-beam interactions and otherwise a linear lattice. Correctors in the arcs were used for a global compensation of long-range beam-beam interactions. In Fig. 1, the ratio of λ_n after to before the compensation was plotted as a function of the order of the compensation for different ξ. In all the cases, after the compensation the quadratic nonlinearity (λ_2) in the one-turn map was reduced to roughly 10% of the original, and higher-order terms ($\lambda_n, n>2$) were also reduced significantly. The compensation is therefore very effective in reducing the nonlinearities due to long-range beam-beam interactions. The reduction rate of λ_n after the compensation, however, decreases with n. As the number of monomials of a given order in the one-turn map increases quickly with the order, it is difficult computationally to locate an optimized minimum (semiglobal minimum), in the minimization of λ_n when n is large. Figure 1 also shows that the reduction rate of λ_n after the compensation is about the same for all ξ.

To study the effect of the multipole compensation of long-range beam-beam interactions, dynamic aperture (DA) was calculated with 10^5-turns element-by-element tracking before and after the compensation. Figure 2 plots the DA as a function of ξ without or with the multipole compensation. When the DA without the compensation is smaller than 9σ, the multipole compensation improves the DA significantly. Without the compensation, the DA also decreases with ξ quickly. After the compensation, such reduction of the DA becomes much slower as more nonlinear is the system, the larger is the increase of the DA after the compensation. In the case that the DA without the compensation is larger than 9σ when $\xi<0.005$, the original system is already quite linear and the compensation fails to further improve the DA. Note that 9σ is the average beam separation at parasitic collisions. In the phase-space region that is close to or larger than the beam separation, the expansions for the long-range beam-beam interaction in Eqs. (6)–(9) are invalid and the multipole correctors for the compensation could make the system even more nonlinear there. The phase-space region that is relevant to the beams is, however, near the closed orbit. In that region, the expansion of the long-range beam-beam interactions is always good and the compensation should therefore be effective in improving the dynamics of the beams. When the original DA is close to or larger than the beam separation, the DA is thus no longer a good quantity to characterize the benefit of the multipole compensation of long-range beam-beam interactions. In Fig. 3, the increase of DA after the compensation was plotted as a function of the order of the compensation. It shows that as the order increases the further improvement of the DA becomes less pronounced. This indicates that the lower-order (cubic and fourth-order) nonlinearities dominate long-range beam-beam interactions.

To examine the effect of the multipole compensation on the linearity of the phase-space region that is relevant to the beams, the evolution of particle distributions was studied by tracking of 5×10^5 particles for each beam up to 10^6 turns. Because of a large beam separation at the parasitic collisions, long-range beam-beam interactions were calculated with the strong-weak formula in Eqs. (2) and (3).

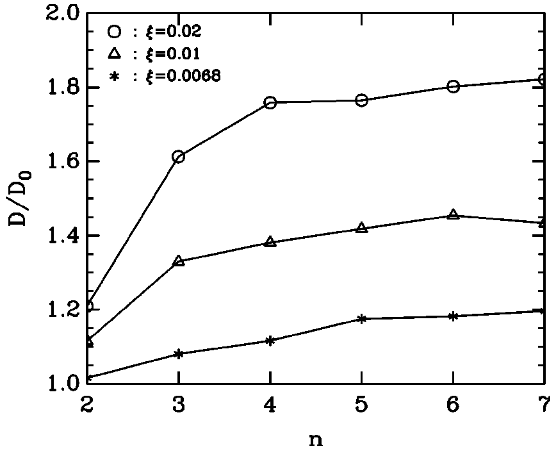

FIG. 3. The increase of dynamic aperture after the multipole compensation of long-range beam-beam interactions vs the order of the compensation. D and D_0 is the dynamic aperture with and without the compensation, respectively. ξ=0.0068, 0.01, or 0.02.

Head-on beam-beam interactions at the IPs, on the other hand, were calculated by using a self-consistent beam-beam simulation code based on the particle-in-cell method [19]. The initial distributions of two beams were round Gaussian in the normalized transverse phase space truncated at 4σ. Without beam-beam interactions, the initial beam distribution matches exactly with the linear lattice. During the simulation, the transverse rms emittance was calculated in terms of $\epsilon=\langle \xi_x^2+\eta_x^2+\xi_y^2+\eta_y^2\rangle/2$, where (ξ_x,ξ_y) and (η_x,η_y) are the normalized transverse coordinates and their conjugate momenta, and $\langle\,\rangle$ is the average over particles in each beam. Because of a large number of particles used in the tracking, the calculated emittance has very low noise (fluctuation due to motions of individual particles). Figure 4 plots the emittance growth without or with the multipole compensation as functions of the beam-beam parameter. It confirms that the multipole compensation of the long-range beam-beam interactions improves the linearity of the phase-space region nearby the closed orbit and, consequently, improves the dynamics of the beams even though the phase-space region close to or beyond the average beam separation at the parasitic crossings could become more nonlinear due to the multipole correctors for the compensation. Moreover, after the multipole compensation the evolution of the emittance with long-range beam-beam interactions was found to almost overlap that without the long-range beam-beam interactions. The multipole compensation therefore eliminates the effect of long-range beam-beam interactions on the emittance growth.

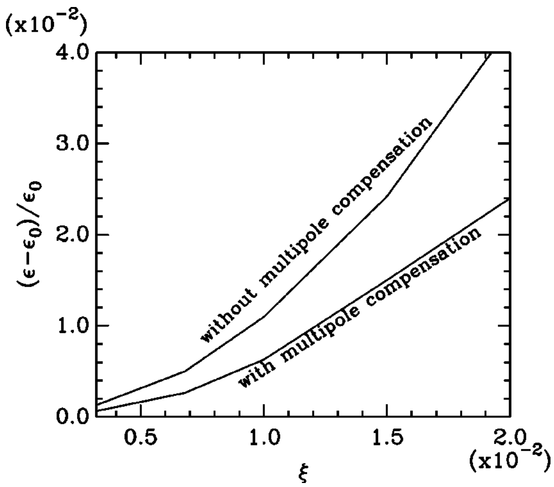

FIG. 4. Emittance growth in 5×10^4 turns vs beam-beam parameter without or with the multipole compensation of long-range beam-beam interactions in the linear lattice with head-on and long-range beam-beam interactions. The upper (lower) curve is without (with) the compensation. ϵ_0 is the initial emittance.

In the case of the nonlinear lattice with beam-beam interactions, the multipole compensation can reduce overall nonlinearities due to both long-range beam-beam interactions and nonlinear field errors in the lattice. To study the effect of the multipole compensation in nonlinear lattice, we have examined 50 different cases of random multiple components of field errors in order to improve the statistical significance of the simulation that involves random magnetic field errors. In this study, the beam-beam parameter was taken to be ξ=0.0068. Note that the nominal beam-beam parameter of LHC is ξ=0.0034. Because in the simulation only two collision points instead of four in LHC were included, we intentionally used ξ that is twice the LHC design value in order to be closer to the LHC situation. The study of DA of those 50 samples showed that without any beam-beam interactions, the smallest and average DA are 7.7σ and 9.8σ, respectively. With both head-on and long-range beam-beam interactions but without the multipole compensation, the smallest and average DA are 5.7σ and 6.5σ. The dynamic aperture of LHC is thus dominated by long-range beam-beam interactions. This result is consistent with other DA studies of LHC [20]. After the fifth-order multipole compensation with the correctors either in the interaction regions or in the arcs, the smallest and average DA increase to 8.2σ and 8.8σ, respectively, which is a more than 40% gain in the DA. Even though the correctors in the arcs are away from the sources of nonlinearities, they are as effective as the correctors inside the IRs for a global compensation of long-range beam-beam interactions and nonlinear field errors. Figure 5 plots the emittance growth with or without the multipole compensation for a typical case of the 50 samples. For a comparison, the emittance growth in the same nonlinear lattice with head-on beam-beam interactions but without long-range beam-beam interactions and without the multipole compensation was also plotted. It shows that without the multipole compensation, the long-range beam-beam interactions result in a 57% increase of the emittance growth during 5×10^4 turns. With the multipole compensation, the increase of the emittance growth due to the long-range beam-beam interactions was again completely eliminated in both cases when the correctors are close to or far away from the parasitic collisions. Moreover, due to a simultaneous compensation of nonlinear fields in the lattice and long-range beam-beam interactions, the emittance growth after the multipole compensation is smaller than that without long-range beam-beam interactions and without the multipole compensation.

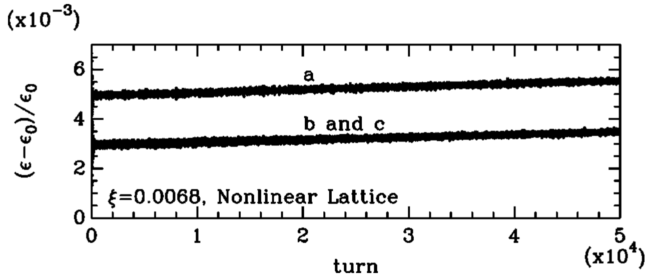

FIG. 5. Evolution of rms transverse emittance in nonlinear lattice with two IPs. ξ=0.0068. (a) With long-range beam-beam interactions but without the multipole compensation. (b) With long-range beam-beam interactions and the multipole compensation. The correctors are in arcs. (c) Without long-range beam-beam interactions and without the multipole compensation. (b) and (c) almost overlap each other and (c) is slightly higher than (b). ϵ_0 is the initial beam emittance.

IV. SUMMARY

The global or local multipole compensation of long-range beam-beam interactions based on the minimization of nonlinearities in one-turn or sectional maps is an effective means to suppress long-range beam-beam effects due to localized or/and distributed parasitic collisions. With a few groups of multipole correctors, nonlinear terms in one-turn or sectional maps including long-range beam-beam interactions can be minimized order by order and, consequently, the nonlinearity of the system in the phase-space region that is relevant to the beams is significantly reduced. In the case of localized parasitic collisions such as in LHC, both the wire compensation [1] and the multipole compensation are very effective in eliminating adverse long-range beam-beam effects. With the multipole compensation, however, the overall nonlinearities in the system including both the long-range beam-beam interactions and magnetic field errors in the lattice can be treated systematically with the same group of multipole correctors since the field errors and long-range beam-beam interactions can be simultaneously considered in the maps. In LHC, for example, there is already a group of multipole correctors inside each inner triplet for a local magnetic-field correction. The multipole compensation of long-range beam-beam interactions could thus be accomplished by using the same group of correctors. In the case of distributed parasitic collisions such as in Tevatron, no other viable solution is currently available for a compensation of nonlinear long-range beam-beam interactions. The multipole compensation scheme proposed here opens up a possibility for the reduction of nonlinear long-range beam-beam effects in this case. To apply the multipole compensation of long-range beam-beam interactions in Tevatron, one needs to consider that the two counterrotating (p and $\overline{p}$) beams of very different intensities have to pass through common multipole correctors because they share a single vacuum pipe. The minimization of nonlinearities of one-turn maps has therefore to be done simultaneously for both the beams. Since the beam-beam interaction on the p beam is much weaker than that on the $\overline{p}$ beam, in the nominal condition of Tevatron RUN II, no compensation of the beam-beam interactions is needed for the p beam. In order to satisfy this unsymmetrical requirement of the multipole strengths of the correctors for the compensation of the long-range beam-beam interactions, the correctors can be placed unsymmetrically with respect to the closed orbits of the two beams, i.e., the orbit of the p beam is at the center of the correctors while the orbit of the $\overline{p}$ beam is away from the center so that the correctors exert much stronger nonlinear perturbations on the $\overline{p}$ beam than that on the p beam. During the minimization of the nonlinearities of the maps, the off-center distance of the $\overline{p}$ beam will be optimized, within any hardware constraints, together with the strengths of the multipole correctors. In the application of the multipole compensation of long-range beam-beam interactions, the strength of correctors have to be reasonable for a fabrication of the correctors. In all the cases we studied, the strengths of the multipoles were found to be in a normal range no matter whether the correctors are close to or away from the parasitic collisions. On the other hand, if the compensation requires stronger correctors, one could increase the number of correctors so that the strength of each corrector can be kept in a preferable range.

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